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TRANSIENT TECHNIQUES FOR BATTERY IMPEDANCE MEASUREMENTS, SMALL --ETC(U)

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Transient Techniques for Battery Impedance Measurements Small-Amplitude Exponential Perturbation Technique

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1 July 1981

Prepared for

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This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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A perturbation technique is reported for measuring the impedance of battery cells under conditions of controlled potential. The small amplitude exponential perturbation (SAEP) technique is applicable over an extremely wide frequency range and appears to be the method of choice for measuring the impedance of battery cells that contain very little stored electrochemical energy.

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CONTENTS

FIGURES

I. INTRODUCTION

The proper operation of battery cells invariably depends on a number of internal physical and chemical reactions occuring at rates that are sufficient to sustain cell performance. These reactions typically involve charge transfer processes at the electrodes, as well as diffusional transport of materials to the active electrode surfaces. Kinetic measurements permit determination of the relative importance of these processes in controlling cell performance. The most general method for making these kinetic measurements is to measure the electrical impedance of the battery cell as a function of frequency. The rates of the various processes that affect the cell voltage are inferred directly from the frequency dispersion of the cell impedance.

A number of techniques have been used to measure the impedance of battery cells. The most commonly used is that of applying a sinusoidally varying ac signal to the battery cell and monitoring the cell response in terms of amplitude and ac phase shift. This ac method is relatively easy to use, but if data are required over a wide frequency range or at very low frequencies, it becomes somewhat cumbersome. Other techniques for impedance measurements of battery cells incorporate perturbing functions other than sinusoidal ac. For example, in the galvanostatic transient technique[1] a step change in the current passing through the cell is applied, and the response of the cell to the current change is measured. The relationship between the change in cell current and the voltage response gives the cell impedance. This technique is particularly useful when the cell contains appreciable stored capacity, since in this case controlling cell current is much easier than controlling cell voltage. However, when the cell contains very little stored capacity, any measurement attempted under conditions of constant current may change the cell voltage by a large amount and thereby appreciably alter the chemical state of the cell. In this situation, it is desirable to employ a potentiostatic

[1]A. H. Zimmerman and M. R. Martinelli, Transient Techniques for Low Frequency Impedance Measurements, TR-0079(4970-10)-1, The Aerospace Corporation, El Segundo, Calif (6 October 1978).

technique that involves the application of a controlled perturbation to the cell potential.

We have developed and applied such a technique to battery cells. This technique is called small amplitude exponential perturbation (SAEP) and involves perturbing the cell voltage with a small amplitude (<5 mV) exponential signal while measuring the current response of the cell. Again, the cell impedance is obtained from the relationship between voltage and current. This technique can be used to measure the impedance of battery cells at any voltage or state of charge that is accessible to them, although very large currents (and power supplies) may be involved when the cell has appreciable active electrochemical capacity.

II. THEORY OF SAEP

Any potentiostatic transient technique for measuring impedance employs a transient potential function V(t). This potential function is applied as a perturbation to a battery cell that has the initial potential V_o. The cell current is initially $I_o + I_N(t)$, where I_o is the steady-state current at V_o, and $I_N(t)$ is any change in current resulting from depletion of the stored electrochemical capacity of the cell at the initial voltage. After the perturbation V(t) is applied, the current is $I_o + I_N(t) + I(t)$. For this analysis to be correct, the amplitude of V(t) must be sufficiently small that $I_N(t)$ does not change appreciably in response to V(t). This means that typically V(t) should be less than 5 mV in amplitude. In addition, the time constant associated with $I_N(t)$ must be much greater than that associated with I(t) so that they can be separated in time.

The impedance as a function of time is then directly given by Ohm's law

$$Z(t) = \frac{V(t)}{I(t)} \qquad (1)$$

However, the cell impedance is more conveniently analyzed in the frequency domain. Laplace transformation of V(t) and I(t) permits us to obtain the impedance as a function of frequency.

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \qquad (2)$$

where $V(\omega)$ and $I(\omega)$ are the Laplace transforms of voltage and current, respectively.

The digital Laplace transforms required are calculated from the voltage and current data,

$$F(\omega) = \int_o^\infty f(t) \exp(-j\omega t)\, dt = \sum_i \int_{t_i}^{t_{i+1}} f_i(t) \exp(-j\omega t) dt \qquad (3)$$

which are digitized by computer into arrays having i data points, each corresponding to a given time. The function $f_i(t)$ fits the data for f(t) in the interval t_i to t_{i+1} and may be any convenient function that fits the data. Functions used for f(t) include linear, quadratic, and exponential forms as follows.

1. Linear: $f_i(t) = A_i t + B_i$

$$A_i = \frac{f(t_i) - B_i}{t_i}$$

$$B_i = \frac{\lfloor f(t_{i+1}) - t_{i+1}/t_i \; f(t_i) \rfloor}{1 - \frac{t_{i+1}}{t_i}} \qquad (4)$$

2. Quadratic: $f_i(t) = L_i t^2 + M_i t + N_i$

$$M_i = \left\lfloor \frac{\Delta f_{12}}{\Delta t_{12}^2} \left(\frac{t_1^2}{t_{i+2}} - t_{i+2} \right) - \frac{\Delta f_{13}}{t_{i+2}} \right\rfloor \left\lfloor 1 - (t_{i+2}) \frac{\Delta t_{12}}{\Delta t_{12}^2} + \frac{t_i}{t_{i+2}} \left(\frac{t_i \Delta t_{12}}{\Delta t_{12}^2} - 1 \right) \right\rfloor^{-1}$$

$$L_i = \frac{\Delta f_{12}}{\Delta t_{12}^2} - M_i \left(\frac{\Delta t_{12}}{\Delta t_{12}^2} \right) \qquad (5)$$

$$N_i = f(t_i) - L_i t_i^2 - M_i t_i$$

where

$$\Delta t_{12} = t_i - t_{i+1}$$

$$\Delta t^2_{12} = t^2_i - t^2_{i+1}$$

$$\Delta f_{12} = f(t_i) - f(t_{i+1})$$

$$\Delta f_{13} = f(t_i) - f(t_{i+2})$$

3. Exponential:

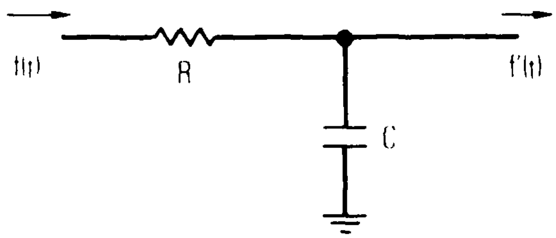
$$f_i(t) = C_i \exp(-a_i t) \qquad (6)$$

$$a_i = \ln \left[\frac{f(t_{i+1})/f(t_i)}{t_i - t_{i+1}}\right]$$

$$C_i = f(t_i) \exp(a_i t_i)$$

The exponential function of Eq. (6) gave the best results for all data that were not near a point where f(t) crossed zero. A listing of a Fortran program for doing the transformations that give the impedance is provided in the Appendix.

Actual experimental data contain noise; in particular, 60-Hz noise may pose a problem when small changes in voltage or current are being monitored. The simplest way to eliminate this kind of noise is with an RC filter.



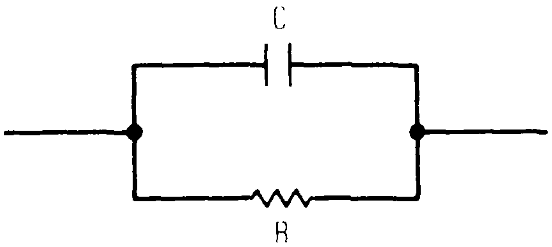
However, the transfer function of this filter must then be deconvoluted from the data. This is easily done in the frequency domain simply by multiplying the transformed function by the inverse filter function transform

$$f(\omega) = f'(\omega)\left(1 + j\omega\tau_F\right) \tag{7}$$

where $\tau_F = RC$ is the filter time constant.

III. RESULTS

The impedance of battery cells below 1 kHz is generally capacitive in nature, behaving as an equivalent parallel RC circuit where the values of R and C may have a complicated dependence on frequency. The results of an SAEP experiment on a simple dummy cell consisting of the RC circuit



where C = 1 F and R = 10 Ω are examined first. For simplicity, let us assume that $V_o + V_N(t)$, the initial cell voltage, is *zero*. An increasing exponential perturbation having amplitude α and time constant τ is applied to the cell

$$V(t) = \alpha\left(1 - e^{-t/\tau}\right) \tag{8}$$

V(t) and the current response of the dummy cell I(t) are indicated in Fig. 1 for τ = 2s, R = 10 Ω, and C = 1 F. I(t) is given by the relationship

$$I(t) = \frac{\alpha}{R}\left[1 - \left(1 - \frac{RC}{\tau}\right)e^{-t/\tau}\right] \tag{9}$$

Note that the values of R and C do not influence the time constant for current decay, but only control the amplitude of the current transient. From the time-dependent voltage and current functions, the impedance is calculated, with the results shown in Fig. 2 in the complex plane. The results in Fig. 2 agree with the theoretical result for the impedance

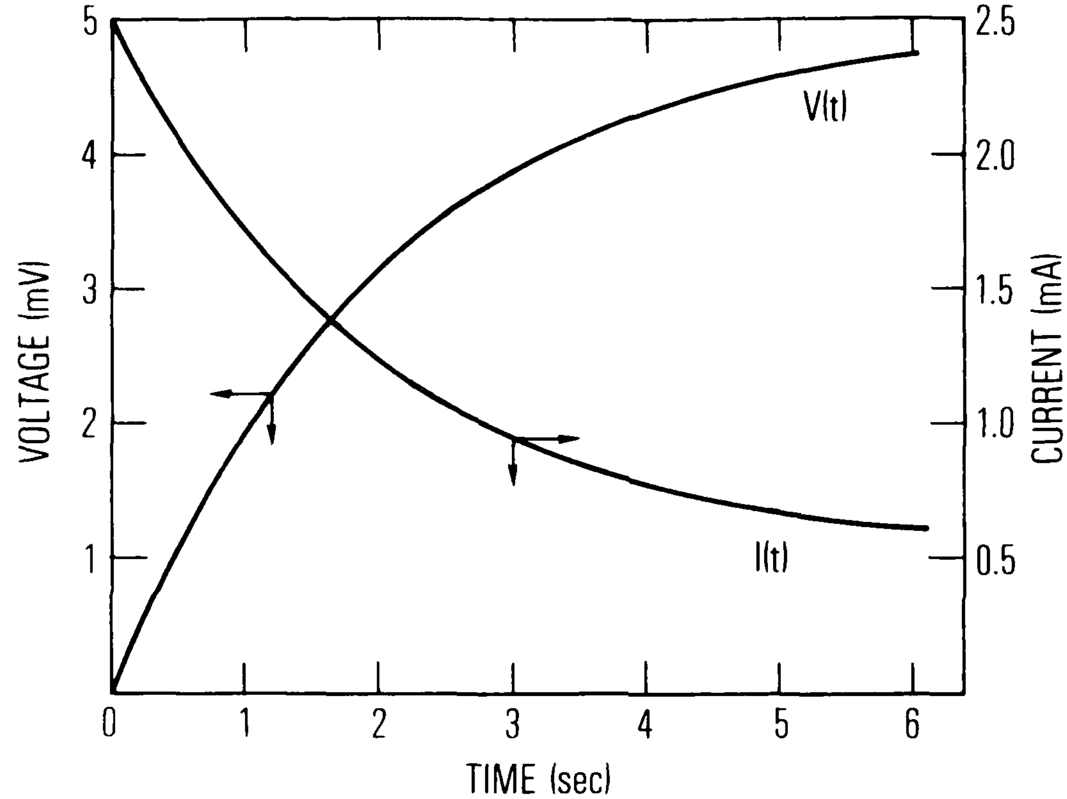


Fig. 1. Voltage Perturbation and Response Current for Dummy Cell

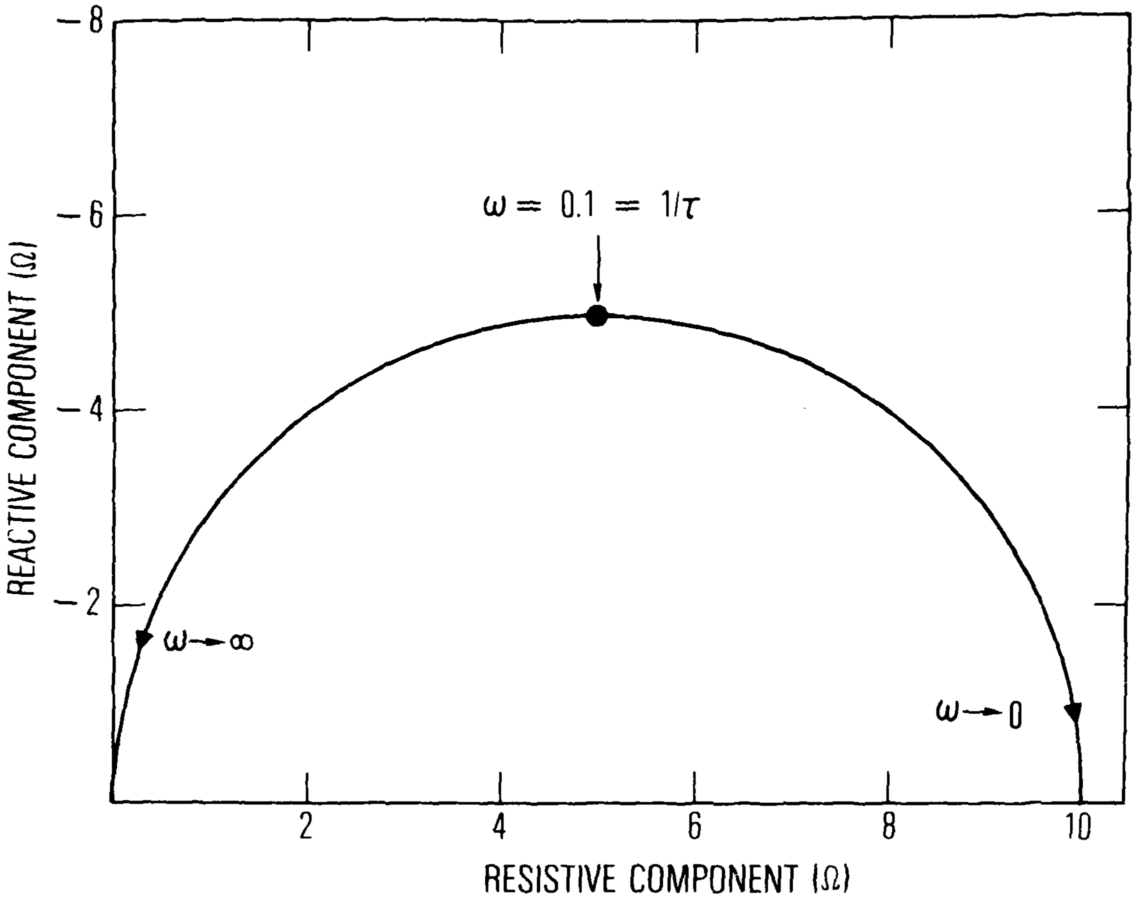


Fig. 2. Impedance of Dummy Cell from Data of Fig. 1

$$Z(\omega) = \frac{R}{1 + j\omega RC} \tag{10}$$

This simple example illustrates that the expected current response for a battery cell consists of a rapid rise to a maximum, followed by a decay to a steady-state current that is different from the initial current by the amount α/R. The magnitude of the peak current is controlled by the relative time constants of the exponential perturbation and the cell, and is given by $\alpha C/\tau$ in the preceding example. Thus, the experimental maximum transient current can be controlled simply by controlling the perturbation time constant.

The results obtained when an exponential perturbation is applied to a nickel cadmium cell are shown in Fig. 3. The nickel cadmium cell used was a 10-Ah prismatic cell, and the initial cell voltage was 0.5 V. The impedance is indicated in the complex plane in Fig. 4. In making these measurements, it was found that signal to noise became relatively poor unless the time constant of the applied perturbation was the same order of magnitude as the relaxation time for the battery cell. With this general requirement satisfied, the SAEP technique provides a convenient method for making impedance measurements on battery cells over an extremely wide range of frequencies, under conditions of battery operation for which potential control is acceptable.

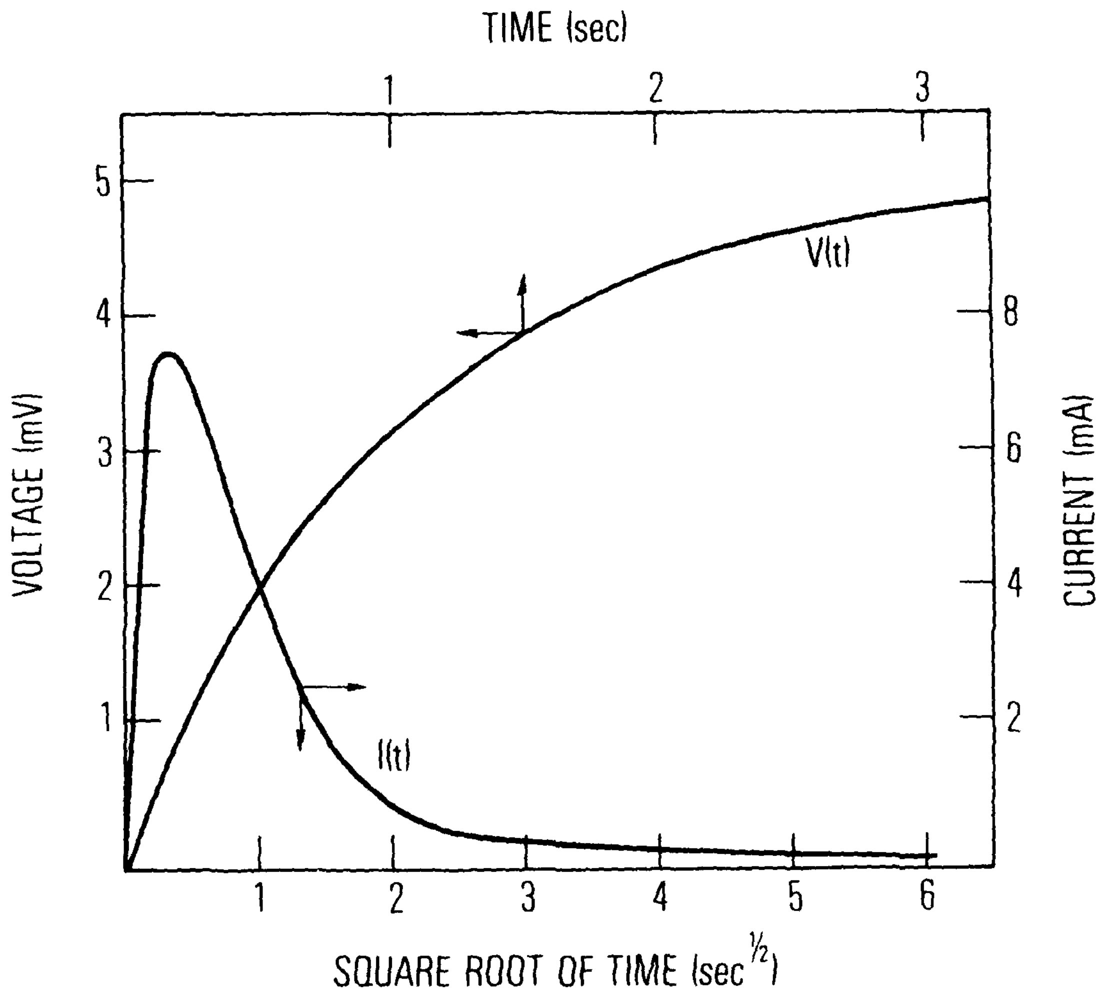


Fig. 3. Voltage Perturbation and Current Response for Nickel Cadmium Cell at 0.5 V

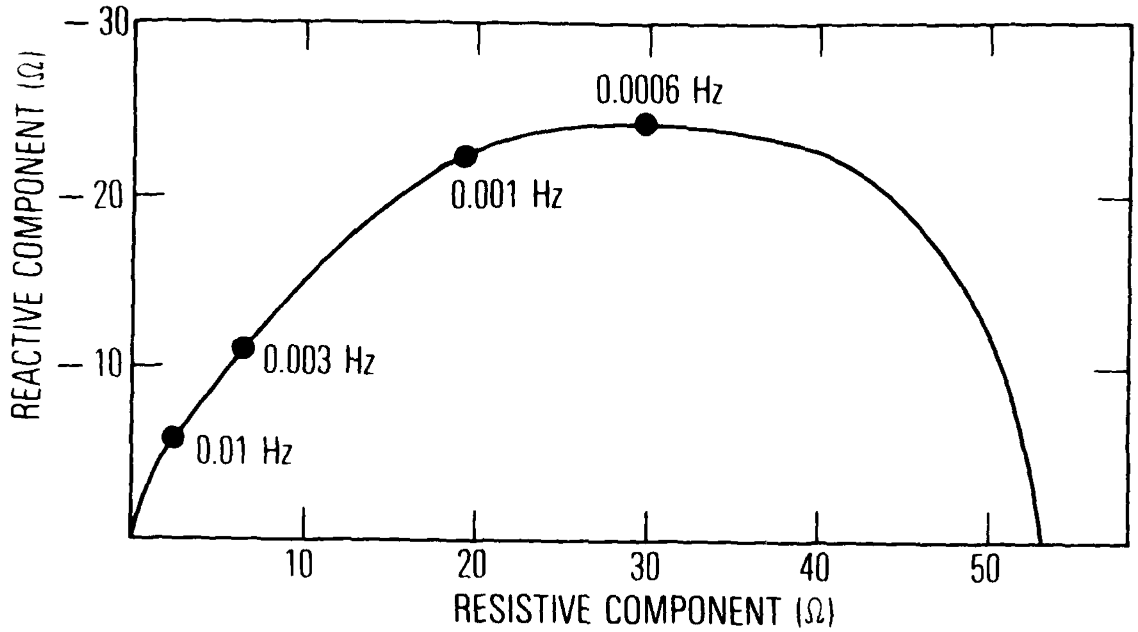


Fig. 4. Impedance of Nickel Cadmium Cell from Data of Fig. 3

IV. CONCLUSIONS

The SAEP technique has been developed and applied to measuring the impedance of battery cells under conditions of controlled potential. This appears to be the optimum method for measuring the impedance of battery cells that contain little stored electrochemical capacity.

APPENDIX

FORTRAN PROGRAM FOR SAEP IMPEDANCE CALCULATION

```
          PROGRAM SAEP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C         THIS PROGRAM CALCULATES IMPEDANCE AS A FUNCTION OF FREQUENCY FROM
C         AN EXPONENTIAL VOLTAGE PERTURBATION AND ITS CURRENT RESPONSE USING
C         THE LAPLACE TRANSFORMATION TECHNIQUE.
C         A FILTER FUNCTION MAY BE INCLUDED IN THE CURRENT RESPONSE, AND IS
C         DECONVOLUTED FROM THE TRANSFORM.
C
C         THE PROGRAM THE FITS THE DATA TO A MAXIMUM OF TWO DIFFUSION
C         PROCESSES. IT REQUIRES INITIAL GUESSES FOR THE FITTING PARAMETERS
C         WHICH ARE TIME CONSTANTS A1 AND A2, WARBURG COEFFICIENTS S1 AND S2
C         AND CAPACITANCES C1 AND C2 AS WELL AS ELECTROLYTE RESISTANCE RZ.
C
C         DATA INPUTS ARE
C         MPROB= EXPERIMENT IDENTIFICATION NUMBER
C              NDATV=NUMBER OF VOLTAGE DATA POINTS
C              NDATI=NUMBER OF CURRENT DATA POINTS
C              TSAEP=NOMINAL PERTURBATION TIME CONSTANT IN SECONDS
C            AMPV=AMPLITUDE OF EXPONENTIAL PERTURBATION IN MV
C             AMPI=AMPLITUDE OF CURRENT CHANGE AT INFINITE TIME IN MA
C            FILTC=TIME CONSTANT OF FILTER USED FOR CURRENT/TIME DATA (SEC)
C            FILTV=TIME CONSTANT OF FILTER USED FOR VOLTAGE/TIME DATA (SEC)
C            VV(I),TSV(I) ARE ORDERED PAIRS CORRESPONDING TO TIME (SEC) AND
C               VOLTAGE DATA (MV), AND SHOULD BE A NOMINALLY INCREASING
C              EXPONENTIAL, ZERO AT ZERO TIME, AND AMPV AT INFINITE TIME.
C             XI(I), TSI(I) ORDERED PAIRS CONSISTING OF THE SQUARE ROOT OF
C              TIME AND CURRENT DATA (MA), AND SHOULD BE ZERO AT ZERO TIME
C              AND XIC AT INFINITE TIME.
          COMMON N,X(200),OBS(400),YMD(800),XS(4000),F(200),RZ
          COMMON/A/A1,S1,C1,A2,S2,C2,VV(200),TSV(200),XI(200),TSI(200)
          COMMON/2/NPROB,PEPS(7)
          COMPLEX ZM(200),VW,XIW,P,XFF,VFF
          EQUIVALENCE (OBS,ZM)
          DATA WINIT,WINC1,WBRK,WINC2,WFIN/0.1,0.4,11.0,1.0,40.0/
C            SWITCH=0 GIVES IMPEDANCE CALCULATION ONLY
C            SWITCH=1 GIVES IMPEDANCE CALCULATION AND FIT TO MODEL
          SWTCH=0
   3  300 CONTINUE
   3      READ(5,2001) MPROB,NDATV,NDATI,TSAEP,AMPV,AMPI,FILTC,FILTV
  27      IF(EOF,5) 1000,301
  32  301 CONTINUE
  32      WRITE(6,6000)
  36      WRITE(6,6001) MPROB
  44      WRITE(6,7000)
  50      WRITE(6,6002) TSAEP,FILTC
  60      WRITE(6,6003) AMPV,AMPI
  70      READ(5,2003)(TSV(I),VV(I),I=1,NDATV)
 107      READ(5,2003)(TSI(I),XI(I),I=1,NDATI)
 126      WRITE(6,7000)
 132      WRITE(6,6004)
 136      DO 302 J=1,NDATV
 140      WRITE(6,6005) J,VV(J),TSV(J)
 153      VV(J)=AMPV-VV(J)
 156  302 TSV(J)=TSV(J)**2
 161      WRITE(6,7000)
 165      WRITE(6,6006)
 171      DO 303 J=1,NDATI
 173      WRITE(6,6007) J,XI(J),TSI(J)
```

```
206        TSI(J)=TSI(J)**2
210   303  XI(J)=XI(J)-AMPI*(1-EXP(-TSI(J)/TSAEP))
223        WRITE(6,7000)
227        N=0
230   320  N=N+1
232        X(N)=WINIT+(N-1)*WINC1
237        IF(X(N).LE.WBRK) GO TO 360
241        NN=N-1
242        X(N)=X(NN)+WINC2
246   360  CONTINUE
250        W=1.0/(X(N)**2)
252        F(N)=W/(2*3.141592654)
254        P=CMPLX(0.,W)
257        CALL LPXFRM(NDATV,TSV,VV,W,VW)
263        CALL LPXFRM(NDATI,TSI,XI,W,XIW)
      C          ADD ZERO FREQUENCY COMPONENT TO TRANSFORM
270        VW=AMPV/P-VW
301        XIW=AMPI/(P*(1+P*TSAEP))+XIW
      C             COMPUTE INVERSE FILTER FUNCTIONS
326        XFF=CMPLX(1.,W*FILTC)
331        VFF=CMPLX(1.,W*FILTV)
      C          COMPUTE INPEDANCE
334        ZM(N)=(VW*VFF)/(XIW*XFF)
361        IF(X(N).GE.WFIN-WINC2) GO TO 401
365        GO TO 320
366   401  CONTINUE
366        WRITE(6,6008)
372        WRITE(6,7000)
376        WRITE(6,6009)
402        DO 420 I=1,N
404        I1=2*I-1
405        I2=I1+1
407        WRITE(6,6010) I,X(I),OBS(I1),OBS(I2),F(I)
430   420  CONTINUE
433        IF(SWTCH.EQ.0.) GO TO 300
434   430  CONTINUE
434        READ(5,2002) A1,S1,C1,A2,S2,C2,RZ
456        N=2*N
457        CALL GAULLL
460        NPROB=1
461        CALL GAUHHH
462        GO TO 300
463  1000  CONTINUE
463        STOP
     2001  FORMAT(3I5,5F10.5)
     2002  FORMAT(7F10.5)
     2003  FORMAT(8F10.5)
     6000  FORMAT(1H1)
     6001  FORMAT(1X,[EXPERIMENT NUMBER [,I5)
     6002  FORMAT(5X,[SAEP TIME CONSTANT =[,2X,F10.5,20X,[FILTER TIME CONSTAN
          *T =[,F10.5,[SECONDS[)
     6003  FORMAT(5X,[VOLTAGE CHANGE IS [,F10.5,20X,[CURRENT CHANGE AT LONG T
          *IMES IS [,F10.5)
     6004  FORMAT(5X,[INPUT DATA  VOLTAGE[,7X,[SQRT TIME[)
     6005  FORMAT(10X,I5,2X,F12.6,2X,F12.6)
     6006  FORMAT(5X,[INPUT DATA  CURRENT[,7X,[SQRT TIME[)
     6007  FORMAT(10X,I5,2X,F12.6,2X,F12.6)
```

```
        6008 FORMAT(1X,[IMPEDANCE DATA[)                                  G        116
        6009 FORMAT(8X,[DATA POINT[,1X,[1/SQRT(OMEGA)[,7X,[REAL  Z[,13X,[IMAGINA G        117
            *RY Z[,8X,[FREQUENCY(HZ)[)                                    G        118
        6010 FORMAT(10X,I5,4(4X,F14.6))                                   G        119
        7000 FORMAT(1X)                                                   G        120
  465        END                                                          G        121


PROGRAM LENGTH INCLUDING I/O BUFFERS
000745

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
300     -  000004     301      -  000033     320      -  000231     360      -  000247
401     -  000357     430      -  000435     1000     -  000464     2001     -  000506
2002    -  000511     2003     -  000513     6000     -  000515     6001     -  000517
6002    -  000524     6003     -  000536     6004     -  000547     6005     -  000555
6006    -  000561     6007     -  000567     6008     -  000573     6009     -  000577
6010    -  000611     7000     -  000615

BLOCK NAMES AND LENGTHS
SAEP    -  000745              -  012742     A        -  001446     2        -  000010

VARIABLE ASSIGNMENTS
AMPI    -  000670     AMPV     -  000667     A1       -  000000S02  A2       -  000003S02
C1      -  000002S02  C2       -  000005S02  F        -  012431S01  FILTC    -  000671
FILTV   -  000672     I        -  000673     I1       -  000677     I2       -  000703
J       -  000674     MPROB    -  000663     N        -  000000S01  NDATI    -  000665
NDATV   -  000664     NN       -  000675     NPROB    -  000000S03  OBS      -  000311S01
P       -  000647     PEPS     -  000001S03  RZ       -  012741S01  SWTCH    -  000662
S1      -  000001S02  S2       -  000004S02  TSAEP    -  000666     TSI      -  001135S02
TSV     -  000316S02  VFF      -  000653     VV       -  000006S02  VW       -  000643
W       -  000676     WBRK     -  000657     WFIN     -  000661     WINC1    -  000655
WINC2   -  000650     WINIT    -  000655     X        -  000001S01  XFF      -  000651
XI      -  000626S02  XIW      -  000645     XS       -  002571S01  YMD      -  001131S01
ZM      -  000311S01

START OF CONSTANTS
000470

START OF TEMPORARIES
000617

START OF INDIRECTS
000635

UNUSED COMPILER SPACE
007400
```

```
      SUBROUTINE GAUHHH
      EXTERNAL MODEL
      COMMON N,X(200),OBS(400),YMD(800),XS(4000),F(200),RZ
      COMMON/A/A1,S1,C1,A2,S2,C2,VV(200),TSV(200),XI(200),TSI(20))
      COMMON/2/NPROB,PEPS(7)
      DIMENSION P(7),BNDUPR(7),BNDLWR(7)
      IS=0
  100 CONTINUE
      NP=7
      P(1)=A1
      P(2)=S1
      P(3)=C1
      P(4)=A2
      P(5)=S2
      P(6)=C2
      P(7)=RZ
      IF(IS.NE.0) GO TO 110
      DO 105 I=1,NP
      BNDUPR(I)=P(I)*1000.
      BNDLWR(I)=P(I)*.001
      PEPS(I)=P(I)*.01
  105 CONTINUE
      EPS1=1.E-6
      EPS2=EPS3=0.
      MAXIT=100
      GAMMA=.01
      FNU=10.
      CALL GAUSAUS(NPROB,MODEL,N,OBS,NP,P,BNDUPR,BNDLWR,EPS1,EPS2,EPS3,
     *MAXIT,GAMMA,FNU,XS)
  110 CONTINUE
      CALL MODEL(NPROB,P,YMD,N,NP)
      U=0.
      S=0.
      WRITE(6,6000)
      DO 120 I=1,N
      ERR=YMD(I)-OBS(I)
      ERR2=ERR**2
      U=U+ERR
      S=S+ERR2
      J=(I+1)/2
      WRITE(6,6001) I,X(J),OBS(I),YMD(I),ERR,ERR2
  120 CONTINUE
      U=U/N
      S=SQRT(S/N)
      WRITE(6,6001)
      WRITE(6,6003) N,U,S
      WRITE(6,6001)
      WRITE(6,6005) (I,P(I),I=1,NP)
      RETURN
      ENTRY GAULLL
      IS=1
      GO TO 100
 6000 FORMAT(1H1)
 6001 FORMAT(1X,I10,3F15.4,2(1PE20.3))
 6003 FORMAT(1X,I10,45X,2(1PE20.3))
 6005 FORMAT(1X,I10,F15.6)
      END
```

```
           SUBROUTINE MODEL(NPROB,Q,G,NG,NQ)
           COMMON N,X(2CC),OBS(40)),YMD(800),XS(4000),F(200),RZ
           COMMON/A/A1,S1,C1,A2,S2,C2,VV(200),TSV(200),XI(200),TSI(20))
           DIMENSION G(1),Q(1)
           COMPLEX ZW1,ZW2,ZC1,ZC2,Z1,Z2,G
           COMPLEX UR,UI
           COMPLEX CTANH
           DATA UR,UI/1.,0.,0.,1./
           A1=Q(1)
   7       S1=Q(2)
  11       C1=Q(3)
  12       A2=Q(4)
  14       S2=Q(5)
  15       C2=Q(6)
  17       RZ=Q(7)
  20       NG2=NG/2
  22       DO 120 I=1,NG2
  25       XX=X(I)
  26       ZC1=-UI*XX**2/C1
  35       ZW1=S1*XX*(1.-UI)*CTANH(A1*CSQRT(UI)/XX)
  70       Z1=ZC1*ZW1/(ZC1+ZW1)
 107       Z2=0.
 110       IF(A2.EQ.0.) GO TO 110
 114       ZC2=-UI*XX**2/C2
 124       ZW2=S2*XX*(1.-UI)*CTANH(A2*CSQRT(UI)/XX)
 156       Z2=ZC2*ZW2/(ZC2+ZW2)
 200   110 CONTINUE
 200       G(I)=Z1+Z2+RZ
 207   120 CONTINUE
 211       RETURN
 212       END


SUBPROGRAM LENGTH
000303

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
110    -  000201

BLOCK NAMES AND LENGTHS
MODEL  -  000303          -  012742    A     -  001446

VARIABLE ASSIGNMENTS
A1     -  000000S02  A2     -  000003S02 CTANH  -  000276     C1     -  000002S02
C2     -  000005S02  F      -  012431S01 I      -  000301     NG2    -  000300
OBS    -  000311S01  RZ     -  012741S01 S1     -  000001S02  S2     -  000004S02
TSI    -  001136S02  TSV    -  000316S02 UI     -  000274     UR     -  000272
VV     -  000006S02  X      -  000001S01 XI     -  000626S02  XS     -  002571S01
XX     -  000302     YMD    -  001131S01 ZC1    -  000262     ZC2    -  000264
ZW1    -  000256     ZW2    -  000260    Z1     -  000266     Z2     -  000270

START OF CONSTANTS
000215
```

```
                  SUBROUTINE PARTIAL(Q,DGDQ,NG,NQ)                                   G         210
                  COMMON N,X(200),OBS(400),YMD(800),XS(4000),F(200),RZ                AJ          2
                  COMMON/2/NPROB,PEPS(7)                                              A2          2
                  DIMENSION Q(1),DGDQ(NG,NQ)                                          G         213
                  DO 120 I=1,NQ                                                       G         214
  10              PEPS12=.5/PEPS(I)                                                   G         215
  11              QSAVE=Q(I)                                                          G         216
  12              Q(I)=QSAVE+PEPS(I)                                                  G         217
  14              CALL MODEL(NPROB,Q,DGDQ(1,I),NG,NQ)                                 G         218
  26              Q(I)=QSAVE-PEPS(I)                                                  G         219
  31              CALL MODEL(NPROB,Q,YMD(1),NG,NQ)                                    G         220
  37              Q(I)=QSAVE                                                          G         221
  44              DO 120 J=1,NG                                                       G         222
  54              DGDQ(J,I)=(DGDQ(J,I)-YMD(J))*PEPS12                                 G         223
  56         120  CONTINUE                                                            G         224
  64              RETURN                                                              G         225
  64              END                                                                 G         226

SUBPROGRAM LENGTH
000105

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

BLOCK NAMES AND LENGTHS
PARTIAL-  000105          -  012742    2      -  000010

VARIABLE ASSIGNMENTS
F      -  012431S01 I      -  000101    J        -  000104     NPROB  -  000101S02
OBS    -  000311S01 PEPS   -  000001S02 PEPS12   -  000102     QSAVE  -  000103
X      -  000001S01 XS     -  002571S01 YMD      -  001131S01

START OF CONSTANTS
000067

START OF TEMPORARIES
000071

START OF INDIRECTS
000075

UNUSED COMPILER SPACE
011200
```

```
                    COMPLEX FUNCTION CTANH(Z)                              G        227
                    COMPLEX UI,7                                           G        228
                    DATA UI/0.,1./                                         G        229
                    IF(CABS(Z).GT.1.) GO T) 1                              G        230
      7             CTANH=-UI*CSIN(UI*Z)/C)OS(UI*Z)                        G        231
     44             RETURN                                                 G        232
     46      1      IF(CABS(Z).GT.39.) GO TO 2                             G        233
     53             CTANH=(1.-CEXP(-2.*Z))/(1.+CEXP(-2.*Z))                G        234
    103             RETURN                                                 G        235
    105      2      CONTINUE                                               G        236
    105             CTANH=CMPLX(1.,0.)                                     G        237
    110             RETURN                                                 G        238
    112             END                                                    G        239


SUBPROGRAM LENGTH
000143

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
1      -   000047     2       -   000106

BLOCK NAMES AND LENGTHS
CTANH  -   000143

VARIABLE ASSIGNMENTS
CTANH  -   000137     UI      -   000141

START OF CONSTANTS
000115

START OF TEMPORARIES
000123

START OF INDIRECTS
000137

UNUSED COMPILER SPACE
011200
```

```
                 SUBROUTINE LPXFRM(NDAT,T,VT,W,VW)
         C       THIS SUBROUTINE COMPUTES THE LAPLACE TRANSFORM AT ANGULAR FREQUENC
         C       W OF THE SET OF ORDERED PAIRS T AND VT CORRESPONDING TO TIME AND
         C       FUNCTION VALUE AT THAT TIME.
         C       VT MUST APPROACH ZERO AT INFINITE TIME AND CANNOT CROSS ZERO.
                 DIMENSION T(200),VT(200)
                 COMPLEX VW,CA,CI,CIP,CA1,CI1,CIP1
                 VW=CMPLX(0.,0.)
   11            IM=NDAT-1
   13            DO 101 I=1,IM
   14            IP=I+1
   15            IF(VT(I).EQ.0.0) VT(I)=0.00001
   21            IF(VT(IP).EQ.0.0) VT(IP)=0.00001
   25            IF(VT(I).LT.0.0) GO TO 150
   27            IF(VT(IP).LT.0.0) GO TO 150
   31            A=ALOG(VT(IP)/VT(I))/(T(I)-T(IP))
   45            CA=CMPLX(A,W)
   47            CI=CMPLX(0.,-W*T(I))
   53            CIP=CMPLX(0.,-W*T(IP))
   57            VW=VW+(VT(I)*CEXP(CI)-VT(IP)*CEXP(CIP))/CA
  120            GO TO 100
  121      150   IF(T(I).EQ.0.0) T(I)=0.00001
  126            DT=T(IP)/T(I)
  130            B=(VT(IP)-DT*VT(I))/(1-DT)
  134            A=(VT(I)-B)/T(I)
  136            CA1=CMPLX(0.,W)
  141            CI1=CEXP(-CA1*T(I))/CA1
  163            CIP1=CEXP(-CA1*T(IP))/CA1
  205            VW=VW+A*(CI1*(T(I)+1./CA1)-CIP1*(T(IP)+1./CA1))+B*(CI1-CIP1)
  264      100   CONTINUE
  267            VW=VW+VT(IP)*CEXP(CIP)/CA
  311      101   CONTINUE
  311            RETURN
  312            END


SUBPROGRAM LENGTH
000420

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
100     -  000265     101     -  000312     150     -  000122

BLOCK NAMES AND LENGTHS
LPXFRM  -  000420

VARIABLE ASSIGNMENTS
A       -  000415     B       -  000417     CA      -  000376     CA1     -  000404
CI      -  000400     CIP     -  000402     CIP1    -  000410     CI1     -  000406
DT      -  000416     I       -  000413     IM      -  000412     IP      -  000414

START OF CONSTANTS
000315
```

DATE

ILMED

-8